

2014年工学部第3問


 数理  
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3 次の問いに答えよ。

(1) 極限值  $\lim_{\alpha \rightarrow 0} \frac{1 - \cos \alpha}{\alpha^2}$  を求めよ。(2)  $\alpha$  を 0 でない実数とすると、定積分  $\int_0^2 (x+1) \cos(\alpha x) dx$  を求めよ。(3) (2) で求めた定積分の値を  $I(\alpha)$  とするとき、極限值  $\lim_{\alpha \rightarrow 0} I(\alpha)$  を求めよ。

$$\begin{aligned}
 (1) \lim_{\alpha \rightarrow 0} \frac{1 - \cos \alpha}{\alpha^2} &= \lim_{\alpha \rightarrow 0} \frac{2}{\alpha^2} \cdot \frac{1 - \cos \alpha}{2} \\
 &= \lim_{\alpha \rightarrow 0} \frac{2}{\alpha^2} \cdot \frac{\sin^2 \frac{\alpha}{2}}{\frac{\alpha^2}{4}} \cdot \frac{\alpha^2}{4} \\
 &= \frac{1}{2} //
 \end{aligned}$$

$$\begin{aligned}
 (2) (\text{与式}) &= \int_0^2 x \left\{ \frac{1}{\alpha} \sin(\alpha x) \right\}' dx + \int_0^2 \cos(\alpha x) dx \\
 &= \left[ \frac{x}{\alpha} \sin(\alpha x) \right]_0^2 - \int_0^2 \frac{1}{\alpha} \sin(\alpha x) dx + \left[ \frac{1}{\alpha} \sin(\alpha x) \right]_0^2 \\
 &= \frac{2}{\alpha} \sin 2\alpha - \frac{1}{\alpha} \left[ -\frac{1}{\alpha} \cos(\alpha x) \right]_0^2 + \frac{1}{\alpha} \sin 2\alpha \\
 &= \frac{3}{\alpha} \sin 2\alpha + \frac{1}{\alpha^2} \cos 2\alpha - \frac{1}{\alpha^2} //
 \end{aligned}$$

$$\begin{aligned}
 (3) \lim_{\alpha \rightarrow 0} I(\alpha) &= \lim_{\alpha \rightarrow 0} \frac{\sin 2\alpha}{2\alpha} \cdot 6 - \frac{2}{\alpha^2} \cdot \sin^2 \alpha \\
 &= \lim_{\alpha \rightarrow 0} \frac{\sin 2\alpha}{2\alpha} \cdot 6 - 2 \cdot \frac{\sin^2 \alpha}{\alpha^2} \\
 &= \frac{4}{1} //
 \end{aligned}$$