

2012年工学部第3問

3 数列 $\{a_n\}$ を

$$a_n = \frac{2n+1}{n(n+1)(n+2)} \quad (n=1, 2, 3, \dots)$$

と定める.

- (1) 定数 p, q を用いて $a_n = p\left(\frac{1}{n} - \frac{1}{n+1}\right) + q\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$ と表すとき, p, q の値を求めよ.
 (2) 数列 $\{a_n\}$ の初項から第 n 項までの和 S_n を求めよ.

$$\begin{aligned} (1) (\text{右辺}) &= p \cdot \frac{1}{n(n+1)} + q \cdot \frac{1}{(n+1)(n+2)} \\ &= \frac{(p+q)n+2p}{n(n+1)(n+2)} \end{aligned}$$

$$\therefore p+q=2 \quad \text{かつ} \quad 2p=1 \quad \therefore p=\frac{1}{2}, q=\frac{3}{2} //$$

$$\begin{aligned} (2) S_n &= \frac{1}{2}\left(\frac{1}{1} - \frac{1}{2}\right) + \frac{3}{2}\left(\frac{1}{2} - \frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{2} - \frac{1}{3}\right) + \frac{3}{2}\left(\frac{1}{3} - \frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{3} - \frac{1}{4}\right) + \frac{3}{2}\left(\frac{1}{4} - \frac{1}{5}\right) + \dots \\ &\quad + \frac{1}{2}\left(\frac{1}{n-1} - \frac{1}{n}\right) + \frac{3}{2}\left(\frac{1}{n} - \frac{1}{n+1}\right) + \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+1}\right) + \frac{3}{2}\left(\frac{1}{n+1} - \frac{1}{n+2}\right) \\ &= \frac{1}{4} + 2\left(\frac{1}{2} - \frac{1}{3}\right) + 2\left(\frac{1}{3} - \frac{1}{4}\right) + 2\left(\frac{1}{4} - \frac{1}{5}\right) + \dots + 2\left(\frac{1}{n} - \frac{1}{n+1}\right) + \frac{3}{2}\left(\frac{1}{n+1} - \frac{1}{n+2}\right) \\ &= \frac{1}{4} + 2\left(\frac{1}{2} - \frac{1}{n+1}\right) + \frac{3}{2}\left(\frac{1}{n+1} - \frac{1}{n+2}\right) \\ &= \frac{n(5n+7)}{4(n+1)(n+2)} // \end{aligned}$$