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中京大学

2016年 工学部（前期M方式）第7問

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$\boxed{7}$ 次の2つの定積分を求めると，

$$\int_0^{\frac{\pi}{2}} 3\sin 3x\,dx = \boxed{\text{ア}}\ 1, \quad \int_0^{\frac{\pi}{2}} tx^2 \sin x\,dx = (\pi - \boxed{\text{イ}}\ 2)t$$

であり，定積分 $\int_0^{\frac{\pi}{2}} \{3\sin 3x - tx^2 \sin x + (t-1)^2\}\,dx$ の最小値は

$$-\frac{\boxed{\text{ウ}}\ 3}{\boxed{\text{エ}}\ 2}\pi - \frac{\boxed{\text{オ}}\ 2}{\pi} + \boxed{\text{カ}}\ 5$$

である．ただし，t は実数とする．

$$\int_0^{\frac{\pi}{2}} 3\sin 3x\,dx = \Big[-\cos 3x\Big]_0^{\frac{\pi}{2}} = \underline{1}\,//$$

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} tx^2 \sin x\,dx &= t\int_0^{\frac{\pi}{2}} x^2(-\cos x)'\,dx \\
&= \underbrace{t\Big[-x^2\cos x\Big]_0^{\frac{\pi}{2}}}_{=0} + t\int_0^{\frac{\pi}{2}} 2x\cos x\,dx \\
&= 2t\int_0^{\frac{\pi}{2}} x(\sin x)'\,dx \\
&= 2t\Big[x\sin x\Big]_0^{\frac{\pi}{2}} - 2t\int_0^{\frac{\pi}{2}} \sin x\,dx \\
&= 2t\left(\frac{\pi}{2}\cdot 1\right) - 2t\Big[-\cos x\Big]_0^{\frac{\pi}{2}} \\
&= \pi t - 2t(0+1) \\
&= \underline{(\pi-2)t}\,//
\end{aligned}$$

$$\begin{aligned}
\therefore \int_0^{\frac{\pi}{2}} \{3\sin 3x - tx^2\sin x + (t-1)^2\}\,dx &= 1 - (\pi-2)t + (t-1)^2\cdot\frac{\pi}{2} \\
&= \frac{\pi}{2}t^2 - 2(\pi-1)t + 1 + \frac{\pi}{2} \\
&= \frac{\pi}{2}\left\{t^2 - \frac{4}{\pi}(\pi-1)t\right\} + 1 + \frac{\pi}{2} \\
&= \frac{\pi}{2}\left\{t - \frac{2}{\pi}(\pi-1)\right\}^2 - \frac{2}{\pi}(\pi-1)^2 + 1 + \frac{\pi}{2} \\
&= \frac{\pi}{2}\left\{t - \frac{2}{\pi}(\pi-1)\right\}^2 - \frac{3}{2}\pi - \frac{2}{\pi} + 5
\end{aligned}$$

∴最小値は，$\underline{-\frac{3}{2}\pi - \frac{2}{\pi} + 5}\,//$