



2016年人文B第5問



5 n を自然数とし, $a_n = \cos n\theta$, $b_n = \sin n\theta$ とする.

(1) a_{n+1} , b_{n+1} を a_n , b_n , $\cos\theta$, $\sin\theta$ を用いて表しなさい.

(2) a_{n+2} を a_{n+1} , a_n , $\cos\theta$ を用いて表しなさい.

(3) $\cos\theta = \frac{3}{4}$ のとき $\cos 5\theta$ の値を求めなさい.

$$(1) \cos(n+1)\theta = \cos n\theta \cos\theta - \sin n\theta \sin\theta$$

$$\therefore a_{n+1} = a_n \cos\theta - b_n \sin\theta \quad \dots \textcircled{1}$$

$$\sin(n+1)\theta = \sin n\theta \cos\theta + \cos n\theta \sin\theta$$

$$\therefore b_{n+1} = b_n \cos\theta + a_n \sin\theta \quad \dots \textcircled{2}$$

$$(2) \textcircled{1} \text{ より } a_{n+2} = a_{n+1} \cos\theta - b_{n+1} \sin\theta \quad \dots \textcircled{3}$$

$$\textcircled{3} - \textcircled{1} \times \cos\theta \text{ より}$$

$$a_{n+2} - a_{n+1} \cos\theta = a_{n+1} \cos\theta - a_n \cos^2\theta - \sin\theta (b_{n+1} - b_n \cos\theta)$$

$$\textcircled{2} \text{ を代入して}$$

$$a_{n+2} - a_{n+1} \cos\theta = a_{n+1} \cos\theta - a_n \cos^2\theta - a_n \sin^2\theta$$

$$\therefore a_{n+2} = 2a_{n+1} \cos\theta - a_n (\cos^2\theta + \sin^2\theta)$$

$$\therefore a_{n+2} = 2a_{n+1} \cos\theta - a_n \quad \dots$$

$$(3) (2) \text{ より } a_3 = 2a_2 \cos\theta - a_1$$

$$a_4 = 2a_3 \cos\theta - a_2$$

$$= 2(2a_2 \cos\theta - a_1) \cos\theta - a_2$$

$$= (4\cos^2\theta - 1)a_2 - 2a_1 \cos\theta$$

$$a_5 = 2a_4 \cos\theta - a_3$$

$$= (8\cos^3\theta - 2\cos\theta)a_2 - 4a_1 \cos^2\theta - (2a_2 \cos\theta - a_1)$$

$$= (8\cos^3\theta - 4\cos\theta)a_2 + (1 - 4\cos^2\theta)a_1$$

$$a_1 = \frac{3}{4}, a_2 = 2 \cdot \left(\frac{3}{4}\right)^2 - 1 = \frac{1}{8} \text{ より } \cos 5\theta = a_5 = -\frac{57}{64} \quad \dots$$