

2015年 神学・経済 第4問


 数理
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4 以下の値を求めよ。

$$(1) \sum_{k=1}^n (2k+1) = \frac{1}{3} n^2 + \frac{2}{3} n$$

$$(2) \sum_{k=1}^n \frac{1}{(2k-1)(2k+1)} = \frac{1}{2} \frac{1}{k} n$$

$$(3) \sum_{k=1}^{2n} (-1)^k 2^{k-1} = \frac{1}{3} (\frac{1}{4} n - 1)$$

$$\begin{aligned} (1) \sum_{k=1}^n (2k+1) &= 2 \sum_{k=1}^n k + \sum_{k=1}^n 1 \\ &= 2 \cdot \frac{1}{2} n(n+1) + n \\ &= \frac{n^2 + 2n}{2} \end{aligned}$$

$$\begin{aligned} (2) \sum_{k=1}^n \frac{1}{(2k-1)(2k+1)} &= \sum_{k=1}^n \frac{1}{2} \left(\frac{1}{2k-1} - \frac{1}{2k+1} \right) \\ &= \frac{1}{2} \left(1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2n-1} - \frac{1}{2n+1} \right) \\ &= \frac{1}{2} \left(1 - \frac{1}{2n+1} \right) \\ &= \frac{n}{2n+1} \end{aligned}$$

$$\begin{aligned} (3) \sum_{k=1}^{2n} (-1)^k \cdot 2^{k-1} &= -1 + 2 - 2^2 + 2^3 - 2^4 + \dots + 2^{2n-1} \quad \left(\begin{array}{l} \text{初項}-1, \text{公比}-2 \text{の} \\ \text{等比数列の和} \end{array} \right) \\ &= \frac{-1 \{1 - (-2)^{2n}\}}{1 - (-2)} \\ &= \frac{1}{3} \{(-2)^{2n} - 1\} \\ &= \frac{1}{3} (4^n - 1) \end{aligned}$$