

2013年第22問


 数理  
石井K

22 関数  $f(x) = \int_1^x (t^2 - t - 6) dt$  の極大値を  $p$ , 極小値を  $q$  とする.  $(pq + 100)$  の値を求めよ.

$$\begin{aligned} f'(x) &= x^2 - x - 6 \\ &= (x-3)(x+2) \end{aligned}$$

$$\therefore f'(x) = 0 \text{ となるのは } x = 3, -2$$

$$\therefore p = f(-2)$$

$$\begin{aligned} &= \int_1^{-2} (t^2 - t - 6) dt \\ &= \left[ \frac{t^3}{3} - \frac{t^2}{2} - 6t \right]_1^{-2} \\ &= -\frac{8}{3} - 2 + 12 - \frac{1}{3} + \frac{1}{2} + 6 \\ &= \frac{27}{2} \end{aligned}$$

$$\therefore pq + 100 = \frac{27}{2} \cdot \left(-\frac{22}{3}\right) + 100$$

$$= 1$$

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$x$	...	-2	...	3	...
$f'(x)$	+	0	-	0	+
$f(x)$	↗		↘		↗

$$q = f(3)$$

$$\begin{aligned} &= \int_1^3 (t^2 - t - 6) dt \\ &= \left[ \frac{t^3}{3} - \frac{t^2}{2} - 6t \right]_1^3 \\ &= 9 - \frac{9}{2} - 18 - \frac{1}{3} + \frac{1}{2} + 6 \\ &= -\frac{22}{3} \end{aligned}$$