自治医科大学

2011年 第9問

9 $\sin\theta+\cos\theta=\frac{1}{2}$ のとき，$(\sin^3\theta-\cos^3\theta)^2$ の値を b とする．$\frac{256b}{25}$ の値を求めよ．

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$(\sin\theta+\cos\theta)^2=\frac{1}{4}$ より． $2\sin\theta\cos\theta=-\frac{3}{4}$ $\therefore \sin\theta\cos\theta=-\frac{3}{8}$

$(\sin^3\theta-\cos^3\theta)^2 = \sin^6\theta+\cos^6\theta-2\sin^3\theta\cos^3\theta$

$= (\underbrace{\sin^2\theta+\cos^2\theta}_{=1})(\sin^4\theta-\sin^2\theta\cos^2\theta+\cos^4\theta)-2(\sin\theta\cos\theta)^3$

$= (\underbrace{\sin^2\theta+\cos^2\theta}_{=1})^2-3(\sin\theta\cos\theta)^2-2(\sin\theta\cos\theta)^3$

$= 1-3\cdot\frac{9}{64}-2\cdot\left(-\frac{27}{512}\right)$

$= 1-\frac{27}{64}+\frac{27}{256}$

$= \frac{256-108+27}{256}$

$= \frac{175}{256}$

$\therefore \frac{256}{25}\times b = \frac{256}{25}\times\frac{175}{256} = 7$ //