

2016年 第3問


 数理
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3 θ が第1象限の角で $\tan\theta + \frac{1}{\tan\theta} = 4$ のとき, $\sin\theta + \cos\theta$ の値を求めよ.

θ : 第1象限の角 より, $\sin\theta > 0, \cos\theta > 0$

よって, $\sin\theta + \cos\theta > 0 \dots \textcircled{1}$

$$\begin{aligned} \tan\theta + \frac{1}{\tan\theta} &= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \\ &= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cos\theta} \\ &= \frac{1}{\sin\theta \cos\theta} \end{aligned}$$

$$\therefore \tan\theta + \frac{1}{\tan\theta} = 4 \text{ より, } \sin\theta \cos\theta = \frac{1}{4} \dots \textcircled{2}$$

$$\begin{aligned} (\sin\theta + \cos\theta)^2 &= \sin^2\theta + 2\sin\theta \cos\theta + \cos^2\theta \\ &= 1 + 2 \cdot \frac{1}{4} \quad (\because \textcircled{2} \text{ より}) \\ &= \frac{3}{2} \end{aligned}$$

$$\therefore \textcircled{1} \text{ より, } \sin\theta + \cos\theta = \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2} //$$