

2016年神学・経済第3問



3 以下の計算をせよ。

$$(1) \sum_{k=0}^8 {}_8C_k = \boxed{\overset{2}{才} \mid \overset{5}{力} \mid \overset{6}{キ}}$$

$$(2) \sum_{k=0}^8 (-1)^k {}_8C_k + \sum_{k=0}^8 {}_8C_k = \boxed{\overset{2}{ク} \mid \overset{5}{ケ} \mid \overset{6}{コ}}$$

$$\begin{aligned} (1) \text{ (手式)} &= {}_8C_0 + {}_8C_1 + {}_8C_2 + {}_8C_3 + {}_8C_4 + {}_8C_5 + {}_8C_6 + {}_8C_7 + {}_8C_8 \\ &= 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 \\ &= \underline{256} \text{ ,,} \end{aligned}$$

(別解) 二項定理より,

$$\sum_{k=0}^n {}_nC_k x^k y^{n-k} = (x+y)^n \quad \cdots \textcircled{1}$$

$$x=y=1, n=8 \text{ を代入して}$$

$$\begin{aligned} \sum_{k=0}^8 {}_8C_k &= 2^8 \\ &= 256 \end{aligned}$$

$$\begin{aligned} (2) \text{ (手式)} &= \sum_{k=0}^8 \{(-1)^k + 1\} {}_8C_k \\ &= 2({}_8C_0 + {}_8C_2 + {}_8C_4 + {}_8C_6 + {}_8C_8) \\ &= 2(1 + 28 + 70 + 28 + 1) \\ &= \underline{256} \text{ ,,} \end{aligned}$$

(別解) ①に $x=-1, y=1, n=8$ を代入して,

$$\sum_{k=0}^8 (-1)^k {}_8C_k = 0$$

$$\begin{aligned} \therefore \text{ (手式)} &= \sum_{k=0}^8 {}_8C_k \\ &= 256 \quad (\because \textcircled{1} \text{ より}) \end{aligned}$$