

2017年 医学部 第7問

増田

7  $A = \frac{1}{1-\alpha} + \frac{1}{1-\alpha^2} + \frac{1}{1-\alpha^3} + \frac{1}{1-\alpha^4} + \frac{1}{1-\alpha^5} + \frac{1}{1-\alpha^6}$  とする.  $\alpha = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}$  であるとき,  $A$  の値を求めよ.

$$\alpha = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7} \text{ より,}$$

$$\alpha^7 = \cos 2\pi + i \sin 2\pi = 1$$

$$\begin{aligned} A &= \left( \frac{1}{1-\alpha} + \frac{1}{1-\alpha^6} \right) + \left( \frac{1}{1-\alpha^2} + \frac{1}{1-\alpha^5} \right) + \left( \frac{1}{1-\alpha^3} + \frac{1}{1-\alpha^4} \right) \\ &= \left( \frac{1}{1-\alpha} + \frac{\alpha}{\alpha - \alpha^7} \right) + \left( \frac{1}{1-\alpha^2} + \frac{\alpha^2}{\alpha^2 - \alpha^7} \right) + \left( \frac{1}{1-\alpha^3} + \frac{\alpha^3}{\alpha^3 - \alpha^7} \right) \\ &= \frac{1-\alpha}{1-\alpha} + \frac{1-\alpha^2}{1-\alpha^2} + \frac{1-\alpha^3}{1-\alpha^3} \\ &= 1 + 1 + 1 \\ &= \underline{\underline{3}} \end{aligned}$$