

2014年人間科学第1問



1 $x + \frac{1}{x} = \sqrt{5}$ のとき、以下の式を完成させよ。

$$(1) x^2 + \frac{1}{x^2} = \overset{3}{\boxed{\text{ア}}}$$

$$(2) x^3 + \frac{1}{x^3} = \overset{2}{\boxed{\text{イ}}} \sqrt{\overset{5}{\boxed{\text{ウ}}}}$$

$$(3) x^5 + \frac{1}{x^5} = \overset{5}{\boxed{\text{エ}}} \sqrt{\overset{5}{\boxed{\text{オ}}}}$$

$$\begin{aligned} (1) x^2 + \frac{1}{x^2} &= \left(x + \frac{1}{x}\right)^2 - 2 \\ &= (\sqrt{5})^2 - 2 \\ &= \underline{3} \text{〃} \end{aligned}$$

$$\begin{aligned} (2) x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right)^3 - 3 \cdot x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) \\ &= (\sqrt{5})^3 - 3 \cdot \sqrt{5} \\ &= 5\sqrt{5} - 3\sqrt{5} \\ &= \underline{2\sqrt{5}} \text{〃} \end{aligned}$$

(別冊)

$$\begin{aligned} x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right) \left(x^2 - 1 + \frac{1}{x^2}\right) \\ &= \sqrt{5} (3 - 1) \\ &= 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} (3) x^5 + \frac{1}{x^5} &= \left(x^2 + \frac{1}{x^2}\right) \left(x^3 + \frac{1}{x^3}\right) - \frac{1}{x} - x \\ &= 3 \cdot 2\sqrt{5} - \sqrt{5} \\ &= \underline{5\sqrt{5}} \text{〃} \end{aligned}$$