

2013年学芸(英文)第2問

2 次の問に答えよ。

- (1) $0 \leq \theta \leq 2\pi$ のとき, $\sin \theta + \cos \theta$ の最大値と最小値を求めよ。
 (2) $0 \leq \theta \leq 2\pi$ のとき, $\sin^3 \theta + \cos^3 \theta$ の最大値と最小値を求めよ。

$$(1) \sin \theta + \cos \theta = \sqrt{2} \sin\left(\theta + \frac{\pi}{4}\right)$$

$$\frac{\pi}{4} \leq \theta + \frac{\pi}{4} \leq \frac{9}{4}\pi \quad \text{よ} \text{)} \quad \underline{\text{最大値は } \sqrt{2} \text{ (} \theta = \frac{\pi}{4} \text{), 最小値は } -\sqrt{2} \text{ (} \theta = \frac{5}{4}\pi \text{)}} //$$

$$(2) t = \sin \theta + \cos \theta \quad \text{と} \text{お} \text{く} \quad (1) \text{よ} \text{)} \quad -\sqrt{2} \leq t \leq \sqrt{2}$$

$$\text{このとき } f(\theta) = \sin^3 \theta + \cos^3 \theta \quad \text{と} \text{お} \text{く}$$

$$t^2 = 1 + 2 \sin \theta \cos \theta \quad \text{よ} \text{)} \quad \sin \theta \cos \theta = \frac{t^2 - 1}{2}$$

$$\begin{aligned} \therefore f(\theta) &= (\sin \theta + \cos \theta)^3 - 3 \sin \theta \cos \theta (\sin \theta + \cos \theta) \\ &= t^3 - 3 \cdot \frac{t^2 - 1}{2} \cdot t \end{aligned}$$

$$= -\frac{1}{2}t^3 + \frac{3}{2}t$$

$$\therefore f'(\theta) = -\frac{3}{2}(t+1)(t-1)$$

t	$-\sqrt{2}$...	-1	...	1	...	$\sqrt{2}$
f'(θ)		-	0	+	0	-	
f(θ)	$-\frac{\sqrt{2}}{2}$	↓	-1	↑	1	↓	$\frac{\sqrt{2}}{2}$

$$\therefore \underline{\text{最大値は } 1 \text{ (} \theta = 0, \frac{\pi}{2} \text{), 最小値は } -1 \text{ (} \theta = \pi, \frac{3}{2}\pi \text{)}} //$$