香川大学

2013年 教育学部・農学部 第4問

$\boxed{4}$ $a > 0$ のとき，2つの放物線 $y = x^2 - 2$，$y = -ax^2 + ax - 1$ について，次の問に答えよ．

(1) 2つの放物線の交点の座標を求めよ．

(2) 2つの放物線で囲まれた図形の面積を求めよ．

(1) $x^2 - 2 - (-ax^2 + ax - 1) = 0$ を解く

$(a+1)x^2 - ax - 1 = 0$

$\begin{array}{ccc} a+1 & & 1 \\ & \times & \\ 1 & & -1 \end{array}$

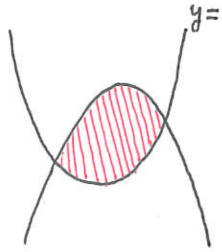
$\therefore (x-1)\{(a+1)x+1\} = 0$

$\therefore x = 1,\ -\dfrac{1}{a+1}$

$x=1$ のとき，$y=-1$，$x = -\dfrac{1}{a+1}$ のとき，$y = \left(-\dfrac{1}{a+1}\right)^2 - 2 = \dfrac{1}{(a+1)^2} - 2$

$\therefore$ 交点は，$\underline{(1, -1),\ \left(-\dfrac{1}{a+1},\ \dfrac{1}{(a+1)^2} - 2\right)}$ //

(2) $\alpha = -\dfrac{1}{a+1}$，$\beta = 1$ とおくと，



$S = \displaystyle\int_\alpha^\beta -ax^2+ax-1-(x^2-2)\,dx$

$= -(a+1)\displaystyle\int_\alpha^\beta (x-1)\left(x+\frac{1}{a+1}\right)dx$

$= -(a+1)\displaystyle\int_\alpha^\beta (x-\alpha)(x-\beta)\,dx$

$\dfrac{1}{6}$公式

$= \dfrac{1}{6}\cdot(a+1)(\beta-\alpha)^3$

$= \dfrac{1}{6}(a+1)\left(1+\dfrac{1}{a+1}\right)^3$

$= \underline{\dfrac{(a+2)^3}{6(a+1)^2}}$ //