

2014年理系2第4問

4 関数 $F(x) = \int_0^{2x} (x-t) \cos 3t dt$ を考える.

$$(1) F'(x) = \frac{\boxed{\text{ク}}}{\boxed{\text{ケ}}} \sin \boxed{\text{コ}} x - \boxed{\text{サ}} x \cos \boxed{\text{シ}} x \text{ より } F'\left(\frac{\pi}{6}\right) = \frac{\boxed{\text{ス}}}{\boxed{\text{セ}}} \frac{\pi}{3} \text{ である.}$$

$$(2) F''(x) = \boxed{\text{ソタ}} x \sin \boxed{\text{チ}} x \text{ より } F''\left(\frac{\pi}{6}\right) = \boxed{\text{ツ}} \text{ である.}$$

$$(1) F(x) = x \int_0^{2x} \cos 3t dt - \int_0^{2x} t \cos 3t dt$$

$$\begin{aligned} \therefore F'(x) &= \int_0^{2x} \cos 3t dt + x \cdot 2 \cdot \cos 6x - 2 \cdot 2x \cos 6x \\ &= \left[\frac{1}{3} \sin 3t \right]_0^{2x} + 2x \cos 6x - 4x \cos 6x \\ &= \frac{1}{3} \sin 6x - 2x \cos 6x \end{aligned}$$

$$F'\left(\frac{\pi}{6}\right) = \frac{1}{3} \sin \pi - \frac{\pi}{3} \cos \pi = \frac{\pi}{3}$$

$$\begin{aligned} (2) (1) \text{ より } F''(x) &= 2 \cos 6x - 2 \cos 6x + 2x \cdot 6 \sin 6x \\ &= 12x \sin 6x \end{aligned}$$

$$\begin{aligned} \therefore F''\left(\frac{\pi}{6}\right) &= 2\pi \cdot \sin \pi \\ &= 0 \end{aligned}$$