# 愛知教育大学

2016年 第9問



9 次の問いに答えよ．

(1) 不定積分 $\int \sin^2 t\,dt$, $\int \sin t \cos t\,dt$, $\int \cos^2 t\,dt$ をそれぞれ求めよ．

(2) 等式

$$f(x) = \cos x + \frac{1}{\pi}\int_0^{\pi} f(t)\cos(t-x)\,dt$$

を満たす $f(x)$ を求めよ．

(1) $\int \sin^2 t\,dt = \int \frac{1-\cos 2t}{2}\,dt = \underline{\frac{t}{2} - \frac{1}{4}\sin 2t + C \ (C\text{は積分定数})}$ //

$\int \sin t\cos t\,dt = \int \frac{1}{2}\sin 2t\,dt = \underline{-\frac{1}{4}\cos 2t + C \ (C\text{は積分定数})}$ //

$\int \cos^2 t\,dt = \int \frac{1+\cos 2t}{2}\,dt = \underline{\frac{t}{2} + \frac{1}{4}\sin 2t + C \ (C\text{は積分定数})}$ //

(2) $f(x) = \cos x + \frac{1}{\pi}\int_0^{\pi} f(t)\cdot(\cos t\cos x + \sin t \sin x)\,dt$

$= \cos x + \frac{1}{\pi}\cdot\cos x\cdot\int_0^{\pi} f(t)\cos t\,dt + \frac{1}{\pi}\cdot\sin x\cdot\int_0^{\pi} f(t)\sin t\,dt$

$a = \int_0^{\pi} f(t)\cos t\,dt$, $b = \int_0^{\pi} f(t)\sin t\,dt$ とおくと，$f(x) = \cos x + \frac{a}{\pi}\cos x + \frac{b}{\pi}\sin x$ … (*)

$\therefore\ a = \int_0^{\pi} \left(1+\frac{a}{\pi}\right)\cos^2 t + \frac{b}{\pi}\sin t\cos t\,dt$

$= \left[\left(1+\frac{a}{\pi}\right)\left(\frac{t}{2}+\frac{1}{4}\sin 2t\right) + \frac{b}{\pi}\left(-\frac{1}{4}\cos 2t\right)\right]_0^{\pi}$

$= \frac{\pi}{2} + \frac{a}{2}$

$\therefore\ a = \frac{\pi}{2} + \frac{a}{2}$ より，$a = \pi$

$b = \int_0^{\pi} \left(1+\frac{a}{\pi}\right)\sin t\cos t + \frac{b}{\pi}\sin^2 t\,dt$

$= \left[\left(1+\frac{a}{\pi}\right)\cdot\left(-\frac{1}{4}\cos 2t\right) + \frac{b}{\pi}\cdot\left(\frac{t}{2}-\frac{1}{4}\sin 2t\right)\right]_0^{\pi}$

$= \frac{b}{2}$

$\therefore\ b = \frac{b}{2}$ より，$b = 0$

(*) に代入して，$\underline{f(x) = 2\cos x}$ //