

2014年医学部第7問



7 次の計算をなさい。

$$\int_0^1 \log(\sqrt{x}+1) dx = \boxed{\frac{1}{2}}, \quad \int_0^1 \left\{ \sqrt{2x-x^2} + \sin\left(x-\frac{1}{2}\right) \right\} dx = \boxed{\frac{\pi}{4}}$$

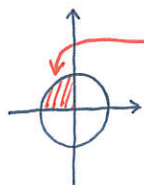
$$t = \sqrt{x} + 1 \text{ とし } \text{置換積分} \text{ する } dt = \frac{dx}{2\sqrt{x}} \Leftrightarrow (2t-2)dt = dx, \quad \begin{array}{l} x \parallel 0 \rightarrow 1 \\ t \parallel 1 \rightarrow 2 \end{array}$$

$$\begin{aligned} \int_0^1 \log(\sqrt{x}+1) dx &= \int_1^2 \log t \cdot (2t-2) dt \\ &= \int_1^2 (t^2-2t)' \log t dt \\ &= \left[(t^2-2t) \log t \right]_1^2 - \int_1^2 (t-2) dt \\ &= - \left[\frac{t^2}{2} - 2t \right]_1^2 \\ &= - \left(2 - 4 - \frac{1}{2} + 2 \right) \\ &= \frac{1}{2} \text{ 〃} \end{aligned}$$

$$t = x-1 \text{ とし } \text{置換積分} \text{ する } dt = dx, \quad \begin{array}{l} x \parallel 0 \rightarrow 1 \\ t \parallel -1 \rightarrow 0 \end{array}$$

$$\int_0^1 \left\{ \sqrt{2x-x^2} + \sin\left(x-\frac{1}{2}\right) \right\} dx = \int_0^1 \sqrt{-(x-1)^2+1} dx + \int_0^1 \sin\left(x-\frac{1}{2}\right) dx$$

$$= \int_{-1}^0 \sqrt{1-t^2} dt + \left[-\cos\left(x-\frac{1}{2}\right) \right]_0^1$$



$$= \frac{\pi}{4} - \cos \frac{1}{2} + \cos\left(-\frac{1}{2}\right)$$

$$= \frac{\pi}{4} \text{ 〃}$$