

2013年 歯学部・薬学部・保健医療 第1問

増田

1 以下の各問に答えよ.

- (1) $6x^2 - 2y^2 + xy - x + 4y - 2$ を因数分解せよ.
 (2) 方程式 $x^2 - x = |x - 2| + 2$ を解け.
 (3) $x = 3 + \sqrt{2}$, $y = 3 - \sqrt{2}$ のとき,
 (i) $x^2 + y^2$, (ii) $x^3 + y^3$, (iii) $x^3 - y^3$
 の値をそれぞれ求めよ.
 (4) $\triangle ABC$ において, $\sin A : \sin B : \sin C = 9 : 7 : 5$ とする. $\sin A$ の値を求めよ.

$$\begin{aligned}
 (1) \quad & 6x^2 - 2y^2 + xy - x + 4y - 2 \\
 &= 6x^2 + (y-1)x - 2(y^2 - 2y + 1) \\
 &= 6x^2 + (y-1)x - 2(y-1)^2 \\
 &= \{2x - (y-1)\} \{3x + 2(y-1)\} \\
 &= \underline{(2x - y + 1)(3x + 2y - 2)}_{\#}
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & x \geq 2 \text{ のとき, } |x-2| = x-2 \\
 & x^2 - x = (x-2) + 2 = x \\
 & x^2 - 2x = 0 \\
 & x(x-2) = 0 \\
 & \therefore x = 0, 2 \\
 & x = 0 \text{ は不適.}
 \end{aligned}$$

$$\begin{aligned}
 & x < 2 \text{ のとき, } |x-2| = -(x-2) \\
 & x^2 - x = -(x-2) + 2 = -x + 4 \\
 & x^2 - 4 = 0 \\
 & (x+2)(x-2) = 0 \\
 & \therefore x = -2, 2 \\
 & x = 2 \text{ は不適.}
 \end{aligned}$$

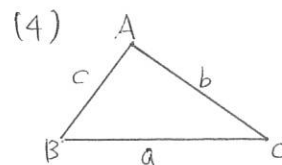
以上より 解は $\underline{x = -2, 2}_{\#}$

$$\begin{aligned}
 (3) \quad & x + y = (3 + \sqrt{2}) + (3 - \sqrt{2}) = 6 \\
 & xy = (3 + \sqrt{2})(3 - \sqrt{2}) = 9 - 2 = 7 \\
 & x - y = (3 + \sqrt{2}) - (3 - \sqrt{2}) = 2\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 (i) \quad & x^2 + y^2 = (x+y)^2 - 2xy \\
 &= 6^2 - 2 \times 7 = \underline{22}_{\#}
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad & x^3 + y^3 = (x+y)(x^2 - xy + y^2) \\
 &= 6 \times (22 - 7) \\
 &= \underline{90}_{\#}
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad & x^3 - y^3 = (x-y)(x^2 + xy + y^2) \\
 &= 2\sqrt{2} \times (22 + 7) \\
 &= \underline{58\sqrt{2}}_{\#}
 \end{aligned}$$



左図のように辺の長さをおく。
 $\sin A = 9k$, $\sin B = 7k$,
 $\sin C = 5k$ と表すと
 ができる (正弦定理より)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$\begin{cases} a = 18kR = 9k' \\ b = 14kR = 7k' \\ c = 10kR = 5k' \end{cases}$$

余弦定理より

$$\cos A = \frac{(7k')^2 + (5k')^2 - (9k')^2}{2 \times 7k' \times 5k'} = -\frac{1}{10}$$

$$\sin A = \sqrt{1 - \cos^2 A} = \frac{\sqrt{99}}{10} = \underline{\frac{3\sqrt{11}}{10}}_{\#}$$