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2013年 第4問　　1枚目/2枚

数理 石井K

$\boxed{4}$ 次の定積分を求めよ.

(1) $\displaystyle\int_2^3 \frac{x^3+2}{x-1}\,dx$　(2) $\displaystyle\int_0^3 e^{\sqrt{x}}\,dx$　(3) $\displaystyle\int_0^{\frac{\pi}{6}} \frac{\log\cos x}{\cos^2 x}\,dx$

$$\begin{array}{r|l} & x^2+x+1 \\ \hline x-1 & x^3+2 \\ & x^3-x^2 \\ \hline & x^2+2 \\ & x^2-x \\ \hline & x+2 \\ & x-1 \\ \hline & 3 \end{array}$$

(1) $\displaystyle\int_2^3 \frac{x^3+2}{x-1}\,dx = \int_2^3 x^2+x+1+\frac{3}{x-1}\,dx$

$\displaystyle = \left[\frac{x^3}{3}+\frac{x^2}{2}+x+3\log(x-1)\right]_2^3$

$\displaystyle = 9+\frac{9}{2}+3+3\log 2-\frac{8}{3}-2-2$

$\displaystyle = \underline{\frac{59}{6}+3\log 2}$ //

(2) $t=\sqrt{x}$ とおいて置換積分 $dt=\dfrac{1}{2\sqrt{x}}dx$，$\begin{array}{c|c} x & 0\to 3 \\ \hline t & 0\to\sqrt{3}\end{array}$

(与式) $\displaystyle = \int_0^{\sqrt{3}} 2te^t\,dt$

$\displaystyle = 2\int_0^{\sqrt{3}} t(e^t)'\,dt$

$\displaystyle = 2\left[te^t\right]_0^{\sqrt{3}} - 2\cdot\int_0^{\sqrt{3}} e^t\,dt$

$\displaystyle = 2(\sqrt{3}e^{\sqrt{3}}) - 2\left[e^t\right]_0^{\sqrt{3}}$

$= 2\sqrt{3}e^{\sqrt{3}} - 2e^{\sqrt{3}} + 2$

$= \underline{2+2(\sqrt{3}-1)e^{\sqrt{3}}}$ //

2013 年 第 4 問　2枚目／2枚

数理 石井K

4 次の定積分を求めよ．

(1) $\displaystyle\int_2^3 \frac{x^3+2}{x-1}\,dx$　(2) $\displaystyle\int_0^3 e^{\sqrt{x}}\,dx$　(3) $\displaystyle\int_0^{\frac{\pi}{6}} \frac{\log\cos x}{\cos^2 x}\,dx$

(3) $(\tan x)' = \dfrac{1}{\cos^2 x}$ より

$$(与式) = \int_0^{\frac{\pi}{6}} (\tan x)' \log\cos x \, dx$$

$$= \Big[\tan x \cdot \log\cos x\Big]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \tan x \cdot \frac{-\sin x}{\cos x}\, dx$$

$$= \frac{1}{\sqrt{3}} \cdot \log\frac{\sqrt{3}}{2} + \int_0^{\frac{\pi}{6}} \tan^2 x \, dx$$

$$= \frac{1}{\sqrt{3}} \log\frac{\sqrt{3}}{2} + \int_0^{\frac{\pi}{6}} \frac{1}{\cos^2 x} - 1 \, dx$$

$$= \frac{1}{\sqrt{3}} \log\frac{\sqrt{3}}{2} + \Big[\tan x - x\Big]_0^{\frac{\pi}{6}}$$

$$= \underline{\frac{1}{\sqrt{3}} \log\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} - \frac{\pi}{6}}\ //$$