

2012年文系第1問

 数理
石井K

 1 $x = \frac{4}{\sqrt{5}-\sqrt{3}}, y = \frac{4}{\sqrt{5}+\sqrt{3}}$ のとき, 次の式の値を求めなさい.

(1) $x^2 + y^2$

(2) $x^2 - y^2$

(3) $x^4 - x^2y^2 + x^2y^4 - y^4$

$$\begin{aligned}
 x+y &= \frac{4}{\sqrt{5}-\sqrt{3}} + \frac{4}{\sqrt{5}+\sqrt{3}} \\
 &= \frac{4(\sqrt{5}+\sqrt{3})+4(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} \\
 &= \frac{8\sqrt{5}}{5-3} \\
 &= 4\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 xy &= \frac{16}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} \\
 &= 8
 \end{aligned}$$

$$\begin{aligned}
 (1) \quad x^2 + y^2 &= (x+y)^2 - 2xy \\
 &= (4\sqrt{5})^2 - 2 \cdot 8 \\
 &= 80 - 16 \\
 &= \underline{64} \text{''}
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad x - y &= \frac{4}{\sqrt{5}-\sqrt{3}} - \frac{4}{\sqrt{5}+\sqrt{3}} \\
 &= \frac{4(\sqrt{5}+\sqrt{3}) - 4(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} \\
 &= 4\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \therefore x^2 - y^2 &= (x+y)(x-y) \\
 &= 4\sqrt{5} \cdot 4\sqrt{3} \\
 &= \underline{16\sqrt{15}} \text{''}
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad x^4 - x^2y^2 + x^2y^4 - y^4 &= x^4 - y^4 - x^2y^2(x^2 - y^2) \\
 &= (x^2 + y^2)(x^2 - y^2) - (xy)^2(x^2 - y^2) \\
 &= 64 \cdot 16\sqrt{15} - 8^2 \cdot 16\sqrt{15} \\
 &= \underline{0} \text{''}
 \end{aligned}$$