

2012年理工B方式第2問


 数理
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2 次の定積分を求めよ。

$$(1) \int_{\frac{1}{2}}^2 x \log x dx = \frac{\boxed{\text{コサ}}^{17}}{\boxed{\text{シ}}^8} \log \boxed{\text{ス}}^2 - \frac{\boxed{\text{セソ}}^{15}}{\boxed{\text{タチ}}^{16}}$$

$$(2) \int_0^2 (x^2 + 2x + 3) \log(x+1) dx = \frac{\boxed{\text{ツテ}}^{15}}{15} \log \boxed{\text{ト}}^3 - \frac{\boxed{\text{ナニ}}^{62}}{\boxed{\text{ヌ}}^9}$$

$$(1) (\#式') = \int_{\frac{1}{2}}^2 \left(\frac{x^2}{2}\right)' \log x dx$$

$$= \left[\frac{x^2}{2} \log x \right]_{\frac{1}{2}}^2 - \int_{\frac{1}{2}}^2 \frac{x}{2} dx$$

$$= 2 \log 2 - \frac{1}{8} \log \frac{1}{2} - \left[\frac{x^2}{4} \right]_{\frac{1}{2}}^2$$

$$= 2 \log 2 + \frac{1}{8} \log 2 - 1 + \frac{1}{16}$$

$$= \frac{17}{8} \log 2 - \frac{15}{16}$$

〃

$$\begin{array}{r} x^2 + 2x + 7 \\ x+1 \overline{) x^3 + 3x^2 + 9x} \\ \underline{x^3 + x^2} \\ 2x^2 + 9x \\ \underline{2x^2 + 2x} \\ 7x + 7 \\ \underline{7x + 7} \\ -7 \end{array}$$

$$(2) (\#式') = \int_0^2 \left(\frac{x^3}{3} + x^2 + 3x\right)' \log(x+1) dx$$

$$= \left[\left(\frac{x^3}{3} + x^2 + 3x\right) \log(x+1) \right]_0^2 - \int_0^2 \frac{\frac{x^3}{3} + x^2 + 3x}{x+1} dx$$

$$= \frac{38}{3} \log 3 - \frac{1}{3} \int_0^2 \frac{x^3 + 3x^2 + 9x}{x+1} dx$$

$$= \frac{38}{3} \log 3 - \frac{1}{3} \int_0^2 (x^2 + 2x + 7) dx + \frac{1}{3} \int_0^2 \frac{7}{x+1} dx$$

$$= \frac{38}{3} \log 3 - \frac{1}{3} \left[\frac{x^3}{3} + x^2 + 7x \right]_0^2 + \frac{1}{3} [7 \log |x+1|]_0^2$$

$$= \frac{38}{3} \log 3 - \frac{62}{9} + \frac{7}{3} \log 3$$

$$= 15 \log 3 - \frac{62}{9}$$

〃

(2)は最初に $t = x+1$ とおいて

置換積分した方が

速いのかも