

2012年第8問


 数理
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8 $\sin \theta + \cos \theta = \frac{1}{\sqrt{5}}$ のとき, $-\frac{8}{13} \left(\tan^3 \theta + \frac{1}{\tan^3 \theta} \right)$ の値を求めよ.

$$(\sin \theta + \cos \theta)^2 = \frac{1}{5} \quad \therefore 2 \sin \theta \cos \theta = -\frac{4}{5}$$

$$\therefore \sin \theta \cdot \cos \theta = -\frac{2}{5}$$

$$\tan^3 \theta + \frac{1}{\tan^3 \theta} = \frac{\sin^3 \theta}{\cos^3 \theta} + \frac{\cos^3 \theta}{\sin^3 \theta}$$

$$= \frac{\sin^6 \theta + \cos^6 \theta}{\sin^3 \theta \cos^3 \theta}$$

$$= \frac{(\sin^2 \theta + \cos^2 \theta)(\sin^4 \theta - \sin^2 \theta \cos^2 \theta + \cos^4 \theta)}{(\sin \theta \cdot \cos \theta)^3}$$

$$= \frac{(\sin^2 \theta + \cos^2 \theta)^2 - 3(\sin \theta \cos \theta)^2}{\left(-\frac{2}{5}\right)^3}$$

$$= \frac{1 - 3 \cdot \left(-\frac{2}{5}\right)^2}{\left(-\frac{2}{5}\right)^3}$$

$$= \frac{1 - \frac{12}{25}}{-\frac{8}{125}}$$

$$= -\frac{125 - 60}{8}$$

$$= -\frac{65}{8}$$

$$\begin{aligned} & \sqrt{\cdot} \\ (\text{与式}) &= -\frac{8}{13} \times \left(-\frac{65}{8}\right) \end{aligned}$$

$$= \underline{\underline{5}}$$