

2010年薬学部第1問

1  $x + \frac{1}{x} = 3$ ,  $y + \frac{1}{y} = 5$  のとき,  $x$  の値は  $\frac{\boxed{3} \pm \sqrt{\boxed{5}}}{\boxed{2}}$ ,  $y$  の値は  $\frac{\boxed{5} \pm \sqrt{\boxed{21}}}{\boxed{2}}$  であり,  $xy + \frac{1}{xy}$  の値は  $\frac{\boxed{15} \pm \sqrt{\boxed{105}}}{\boxed{2}}$  である.

$$x + \frac{1}{x} = 3 \text{ より, } x^2 - 3x + 1 = 0 \quad \therefore x = \frac{3 \pm \sqrt{5}}{2}$$

$$y + \frac{1}{y} = 5 \text{ より, } y^2 - 5y + 1 = 0 \quad \therefore y = \frac{5 \pm \sqrt{21}}{2}$$

$$\frac{1}{x} = 3 - x \text{ より, } \frac{1}{x} = 3 - \frac{3 \pm \sqrt{5}}{2} = \frac{3 \mp \sqrt{5}}{2}$$

$$\frac{1}{y} = 5 - y \text{ より, } \frac{1}{y} = 5 - \frac{5 \pm \sqrt{21}}{2} = \frac{5 \mp \sqrt{21}}{2}$$

$$\begin{aligned} \therefore xy + \frac{1}{xy} &= \frac{3 \pm \sqrt{5}}{2} \cdot \frac{5 \pm \sqrt{21}}{2} + \frac{3 \mp \sqrt{5}}{2} \cdot \frac{5 \mp \sqrt{21}}{2} && \left( \begin{array}{l} \text{~~~~, ~~~~ はそれぞれ根号同順} \\ \text{それ以外は任意} \end{array} \right) \\ &= \frac{15 \pm 3\sqrt{21} \pm 5\sqrt{5} \pm \sqrt{105}}{4} + \frac{15 \mp 3\sqrt{21} \mp 5\sqrt{5} \mp \sqrt{105}}{4} \\ &= \frac{30 \pm 2\sqrt{105}}{4} \\ &= \frac{15 \pm \sqrt{105}}{2} \end{aligned}$$

