琉球大学

2017年 理系 第2問

$\boxed{2}$ 次の問いに答えよ.

(1) $0 < \theta < \frac{\pi}{2}$ のとき，方程式 $2\sin\theta = \sin 3\theta$ を満たす θ の値を求めよ.

(2) 定積分 $\int_0^{\frac{\pi}{2}} |\sin 3\theta - 2\sin\theta|\, d\theta$ を求めよ.

(1) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ より

$$2\sin\theta = \sin 3\theta \iff \sin\theta - 4\sin^3\theta = 0$$
$$\iff \sin\theta(1+2\sin\theta)(1-2\sin\theta) = 0$$

$\therefore \sin\theta = 0, \pm\frac{1}{2}$

$0 < \theta < \frac{\pi}{2}$ より. $0 < \sin\theta < 1$ $\therefore \sin\theta = \frac{1}{2}$ $\therefore \underline{\theta = \frac{\pi}{6}}$ //

(2) (1)より.

$0 \leqq \theta \leqq \frac{\pi}{6}$ において. $\sin 3\theta - 2\sin\theta \geqq 0$

$\frac{\pi}{6} \leqq \theta \leqq \frac{\pi}{2}$ において. $\sin 3\theta - 2\sin\theta \leqq 0$

$$\therefore (与式) = \int_0^{\frac{\pi}{6}} \sin 3\theta - 2\sin\theta\, d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} -\sin 3\theta + 2\sin\theta\, d\theta$$

$$= \left[-\frac{1}{3}\cos 3\theta + 2\cos\theta\right]_0^{\frac{\pi}{6}} + \left[\frac{1}{3}\cos 3\theta - 2\cos\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \sqrt{3} - \left(-\frac{1}{3} + 2\right) - (-\sqrt{3})$$

$$= \underline{2\sqrt{3} - \frac{5}{3}}$$ //