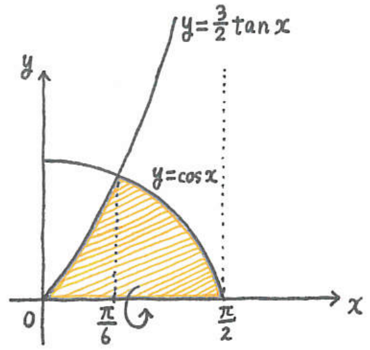
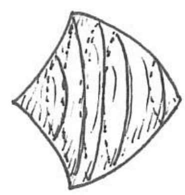
2016年 医学部 第2問

$\boxed{2}$ 2つの曲線 $y=\dfrac{3}{2}\tan x$ $\left(0 \leqq x < \dfrac{\pi}{2}\right)$, $y=\cos x$ $\left(0 \leqq x \leqq \dfrac{\pi}{2}\right)$ と x 軸で囲まれた図形を，x 軸のまわりに1回転してできる立体の体積を求めよ．





2曲線の交点の x 座標を求める．

$$\frac{3}{2}\tan x=\cos x \iff 3\sin x = 2\cos^2 x$$
$$\iff 2\sin^2 x+3\sin x-2=0$$
$$\iff (2\sin x-1)(\sin x+2)=0$$
$$\iff \sin x=\frac{1}{2}$$

$0 \leqq x < \frac{\pi}{2}$ より．

$$x=\frac{\pi}{6}$$

$$\therefore V=\pi\int_0^{\frac{\pi}{6}}\left(\frac{3}{2}\tan x\right)^2 dx+\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\cos^2 x\,dx$$
$$=\frac{9}{4}\pi\int_0^{\frac{\pi}{6}}\tan^2 x\,dx+\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{1+\cos 2x}{2}\,dx$$
$$=\frac{9}{4}\pi\int_0^{\frac{\pi}{6}}\left(\frac{1}{\cos^2 x}-1\right)dx+\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\frac{1}{2}+\frac{1}{2}\cos 2x\right)dx$$
$$=\frac{9}{4}\pi\Big[\tan x-x\Big]_0^{\frac{\pi}{6}}+\pi\left[\frac{x}{2}+\frac{1}{4}\sin 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$
$$=\frac{9}{4}\pi\left(\frac{1}{\sqrt{3}}-\frac{\pi}{6}\right)+\pi\left(\frac{\pi}{4}-\frac{\pi}{12}-\frac{1}{4}\cdot\frac{\sqrt{3}}{2}\right)$$
$$=\underline{\frac{5\pi(3\sqrt{3}-\pi)}{24}}\,/\!/$$