

2011年第3問


 数理
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3 $x^{\frac{1}{4}} + x^{-\frac{1}{4}} = 3$ ($x > 0$, x は実数) のとき, $\frac{47}{2} \left(\frac{x^{\frac{3}{4}} + x^{-\frac{3}{4}}}{x + x^{-1}} \right)$ の値を求めよ.

$$\begin{aligned}
 \frac{47}{2} \left(\frac{x^{\frac{3}{4}} + x^{-\frac{3}{4}}}{x + x^{-1}} \right) &= \frac{47}{2} \cdot \frac{(x^{\frac{1}{4}} + x^{-\frac{1}{4}})(x^{\frac{1}{2}} - 1 + x^{-\frac{1}{2}})}{(x^{\frac{1}{2}} + x^{-\frac{1}{2}})^2 - 2} \\
 &= \frac{47}{2} \cdot \frac{3 \cdot \{(x^{\frac{1}{4}} + x^{-\frac{1}{4}})^2 - 2 - 1\}}{\{(x^{\frac{1}{4}} + x^{-\frac{1}{4}})^2 - 2\}^2 - 2} \\
 &= \frac{47}{2} \cdot \frac{3(9-3)}{7^2 - 2} \\
 &= \frac{47}{2} \cdot \frac{18}{47} \\
 &= \underline{\underline{9}}
 \end{aligned}$$