

2013年理系2第5問

5 行列 $A = \frac{1}{2} \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ を考える. また, E を単位行列とする.

(1) $A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ ($0 \leq \theta < 2\pi$) と表すと, $\theta = \frac{\boxed{\text{ア}}}{\boxed{\text{イ}}} \frac{\pi}{\boxed{\text{エ}}}$ である.

(2) $E + A + A^2 = \begin{pmatrix} \boxed{\text{ウ}} \boxed{1} & -\sqrt{\boxed{\text{エ}} \boxed{3}} \\ \sqrt{\boxed{\text{オ}} \boxed{3}} & \boxed{\text{カ}} \boxed{1} \end{pmatrix}$, $A^3 = \begin{pmatrix} \boxed{\text{キ}} \boxed{\text{ク}} & \boxed{\text{ケ}} \\ \boxed{\text{コ}} \boxed{0} & \boxed{\text{サ}} \boxed{\text{シ}} \end{pmatrix}$, $E + A + A^2 + A^3 + A^4 + A^5 = \begin{pmatrix} \boxed{\text{ス}} \boxed{0} & \boxed{\text{セ}} \boxed{0} \\ \boxed{\text{ソ}} \boxed{0} & \boxed{\text{タ}} \boxed{0} \end{pmatrix}$ である.

(3) $E + A + A^2 + A^3 + \dots + A^{20} = \begin{pmatrix} \boxed{\text{チ}} \boxed{1} & -\sqrt{\boxed{\text{ツ}} \boxed{3}} \\ \sqrt{\boxed{\text{テ}} \boxed{3}} & \boxed{\text{ト}} \boxed{1} \end{pmatrix}$ である.

(1) $A = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ $\therefore \cos \theta = \frac{1}{2}$, $\sin \theta = \frac{\sqrt{3}}{2}$ $0 \leq \theta < 2\pi$ より, $\theta = \frac{\pi}{3}$ „

(2) $A^2 = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ $\therefore E + A + A^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ „

$$A^3 = A \cdot A^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$E + A + A^2 + A^3 + A^4 + A^5 = (E + A^3)(E + A + A^2) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (\because E + A^3 = O)$$

$$\begin{aligned} (3) E + A + A^2 + A^3 + \dots + A^{20} &= (E + A^6 + A^{12})(E + A + A^2 + A^3 + A^4 + A^5) + A^{18} + A^{19} + A^{20} \\ &= A^{18} + A^{19} + A^{20} \\ &= (A^3)^6 \cdot (E + A + A^2) \\ &= (-E)^6 \cdot \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix} \end{aligned}$$