

2013年工学部第4問



4 関数 $f(x) = x^2 e^{-x}$ について以下の問いに答えよ.

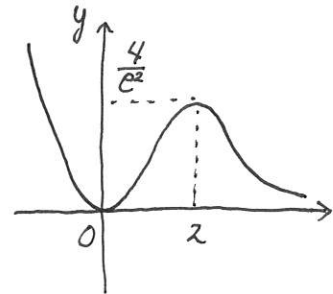
- (1) $f'(x)$ を求めよ.
 (2) $f(x)$ の極値を求めグラフの概形を描け (変曲点は求めなくてよい).
 (3) $\int_0^1 f(x) dx$ を求めよ.

$$\begin{aligned}
 (1) \quad f'(x) &= 2x e^{-x} + x^2 \cdot (-e^{-x}) \\
 &= \underline{x(2-x)e^{-x}}
 \end{aligned}$$

x	...	0	...	2	...
$f'(x)$	-	0	+	0	-
$f(x)$	↓	0	↑	$\frac{4}{e^2}$	↓
				極小	極大

$$\begin{aligned}
 (2) \quad &\left\{ \begin{array}{l} \text{極小値 } 0 \text{ (} x=0 \text{ のとき)} \\ \text{極大値 } \frac{4}{e^2} \text{ (} x=2 \text{ のとき)} \end{array} \right. \\
 &\underline{\hspace{10em}}
 \end{aligned}$$

$$\left(\lim_{x \rightarrow -\infty} f(x) = +\infty, \lim_{x \rightarrow \infty} f(x) = 0 \right)$$



$$\begin{aligned}
 (3) \quad \int_0^1 f(x) dx &= \int_0^1 x^2 (-e^{-x})' dx \\
 &= [-x^2 e^{-x}]_0^1 - \int_0^1 -2x e^{-x} dx \\
 &= -\frac{1}{e} - 2 \int_0^1 x (e^{-x})' dx \\
 &= -\frac{1}{e} - 2 [x e^{-x}]_0^1 + 2 \int_0^1 e^{-x} dx \\
 &= -\frac{1}{e} - 2 \cdot \left(\frac{1}{e}\right) + 2 [-e^{-x}]_0^1 \\
 &= -\frac{1}{e} - \frac{2}{e} - 2 \cdot \frac{1}{e} + 2 \\
 &= \underline{2 - \frac{5}{e}}
 \end{aligned}$$