山形大学

3 次の問に答えよ.

(1) 不定積分 $\displaystyle\int t\sin t\,dt$ を求めよ.

(2) 定積分 $\displaystyle\int_0^{\frac{\pi}{2}}\left|\frac{2}{3}\pi-2t\right|\sin t\,dt$ を求めよ.

(3) 関数 $f(x)$ を $\displaystyle f(x)=\int_0^{\frac{\pi}{2}}|x-2t|\sin t\,dt$ で定める（$0\leqq x\leqq\pi$）. $f(x)$ の最大値, 最小値を求め, それらを与える x の値をそれぞれ求めよ.

(1) $\displaystyle\int t\sin t\,dt = \int t(-\cos t)'\,dt$

$\displaystyle\qquad = -t\cos t - \int -\cos t\,dt$

$\displaystyle\qquad = \sin t - t\cos t + C \quad (C\text{ は積分定数})$

(2) $\displaystyle\int_0^{\frac{\pi}{2}}\left|\frac{2}{3}\pi-2t\right|\sin t\,dt = \int_0^{\frac{\pi}{3}}\left(\frac{2}{3}\pi-2t\right)\sin t\,dt + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\left(2t-\frac{2}{3}\pi\right)\sin t\,dt$

$\displaystyle\qquad = \frac{2}{3}\pi\int_0^{\frac{\pi}{3}}\sin t\,dt - 2\int_0^{\frac{\pi}{3}}t\sin t\,dt + 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}t\sin t\,dt - \frac{2}{3}\pi\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\sin t\,dt$

$\displaystyle\qquad = \frac{2}{3}\pi\Big[-\cos t\Big]_0^{\frac{\pi}{3}} - 2\Big[\sin t - t\cos t\Big]_0^{\frac{\pi}{3}} + 2\Big[\sin t - t\cos t\Big]_{\frac{\pi}{3}}^{\frac{\pi}{2}} - \frac{2}{3}\pi\Big[-\cos t\Big]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$\displaystyle\qquad = \frac{2}{3}\pi - 2\sqrt{3} + 2$

(3) $\displaystyle f(x) = \int_0^{\frac{x}{2}}(x-2t)\sin t\,dt + \int_{\frac{x}{2}}^{\frac{\pi}{2}}(2t-x)\sin t\,dt$

$\displaystyle\qquad = x\int_0^{\frac{x}{2}}\sin t\,dt - 2\int_0^{\frac{x}{2}}t\sin t\,dt + 2\int_{\frac{x}{2}}^{\frac{\pi}{2}}t\sin t\,dt - x\int_{\frac{x}{2}}^{\frac{\pi}{2}}\sin t\,dt$

$\displaystyle\therefore f'(x) = \int_0^{\frac{x}{2}}\sin t\,dt + \frac{x}{2}\sin\frac{x}{2} - 2\left(\frac{x}{2}\sin\frac{x}{2}\right)\cdot\frac{1}{2} + 2\cdot\frac{x}{2}\cdot\sin\frac{x}{2}\cdot\left(-\frac{1}{2}\right) - \int_{\frac{x}{2}}^{\frac{\pi}{2}}\sin t\,dt$

$\displaystyle\qquad\qquad -x\cdot\sin\frac{x}{2}\cdot\left(-\frac{1}{2}\right)$

$\displaystyle\qquad = 1 - 2\cos\frac{x}{2}$

$\therefore f'(x)=0$ となる x は. $x=\frac{2}{3}\pi$

$\displaystyle f(0) = 2\int_0^{\frac{\pi}{2}}t\sin t\,dt = 2$

$\displaystyle f\left(\frac{2}{3}\pi\right) = \frac{2}{3}\pi - 2\sqrt{3} + 2$

$\displaystyle f(\pi) = \pi\int_0^{\frac{\pi}{2}}\sin t\,dt - 2 = \pi - 2$

x	0	$\cdots$	$\frac{2}{3}\pi$	$\cdots$	π
$f'(x)$		$-$		$+$	
$f(x)$	2	$\searrow$		$\nearrow$	$\pi-2$

$\therefore$ 最大値 2 $(x=0)$, 最小値 $\frac{2}{3}\pi - 2\sqrt{3} + 2$ $\left(x=\frac{2}{3}\pi\right)$